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$$CD = \frac{ab^2}{b^2+c^2}, AE = \frac{bc^2}{a^2+c^2}, BF = \frac{a^2c}{a^2+b^2};$$
$$\therefore a = \frac{b^2}{b^2+c^2}, \beta = \frac{c^2}{a^2+c^2}, \gamma = \frac{a^2}{a^2+b^2};$$
$$\therefore \frac{A'}{A} = \frac{2a^2b^2c^2}{(a^2+b^2)(a^2+c^2)(b^2+c^2)}.$$

Compare this result with that in 2.

5. Let the points D, E, F , be the feet of the perpendiculars let fall from the centre of the inscribed circle.

Denote the radius of the inscribed circle by ρ and put $\frac{1}{2}(a+b+c) = s$; then $a = (\rho \div a) \cot \frac{1}{2}C$, $\beta = (\rho \div b) \cot \frac{1}{2}A$, $\gamma = (\rho \div c) \cot \frac{1}{2}B$. Therefore

$$\frac{A'}{A} = \frac{2\rho^3}{abc} \cot \frac{1}{2}A \cot \frac{1}{2}B \cot \frac{1}{2}C = \frac{\rho^2 s}{abc} = \frac{2(s-a)(s-b)(s-c)}{abc}$$
$$= \frac{(a+b-c)(a+c-b)(b+c-a)}{4abc}.$$

Compare this result with that in 3.

FIVE GEOMETRICAL PROPOSITIONS.

BY PROF. ELIAS SCHNEIDER, MILTON, PA.

I. LET A, B, C, D , &c., be the angular points of a regular polygon of n sides, and let AB , one of the equal sides, aqual unity; then will AB be contained once in AC , the chord which contains two of the equal sides, with a remainder which call x . Then is $\sqrt{1-x} =$ one side of a polygon of $2n$ sides inscribed in a circle whose radius is *one*.

II. AB will be contained twice in AD , the chord which contains three of the equal sides, with a remainder which call y . Then is $\sqrt{1-y} =$ one side of a polygon of n sides inscribed in a circle whose radius is *one*.

III. If the polygon be a Nonagon, AB will be contained twice in AE , the chord which contains four of the equal sides, with a remainder which call x . Then is $\sqrt{1-x} =$ one side of a polygon of 18 sides inscribed in a circle whose radius is *one*.

IV. If in Prop. II the polygon be also a Nonagon, then is

$$\sqrt{1-x} = x - y.$$

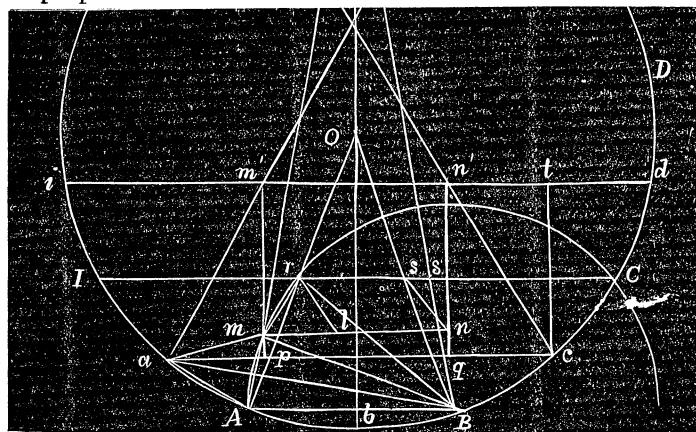
V. If the polygon be a Decagon, AB will be contained twice in AD , the chord which contains three of the equal sides, with a remainder which call z . Then is $\sqrt{1-z} = z =$ one side of a decagon inscribed in a circle whose radius is *one*.

[As the foregoing propositions are new, or at least original with Prof. Schneider (see *ANALYST*, Vol. I, p. 37), and have never been demonstrated geometrically, so far as we know; and as Prof. S. declined to submit his *proof*, we have attempted their demonstration, and submit the following sketch of our method and result.—Ed.]

It will be seen that the Figure is drawn for a Nonagon and that ac , IC , and id are chords containing 2, 3 and 4 of the equal sides, respectively.

The $\angle AOB = ABr = 2AFB = 2ABm$. Draw mn parallel, and mp and nq perpendicular to ac . Then, because aB is perpendicular to Am , we may easily prove that $am = aA$, and $ap = qc = \frac{1}{2}AB = \frac{1}{2}$; therefore $mn = pq = x$, of prop. I.

By Euclid,
Prop. D, B'k
VI, we have
 $Am \times Bn +$
 $AB \times mn =$
 $An \times Bm$; or,
because $Bn =$
 Am , $Am^2 + x$
 $= 1$. Trans-
pos'g and ex-
tracting root
we have Am
 $= \sqrt{(1-x)}$,



which proves Prop. I. And in like manner, substituting rs ($= y$) for mn , we get $Ar = \sqrt{1 - y}$, which proves Prop. II. The proof in both these cases is obviously independent of any particular value of n .

If the polygon is a Nonagon, ch will intersect id at an angle of 60° and therefore cdn' is an equilateral triangle and ct is equal and parallel to qn ; therefore $td = n't = qc = \frac{1}{2}$; therefore $n'd = 1$, and $m'n' = mn = x$, and the demonstration of I applies also to III.

Join sn and draw rl parallel to sn , then, if the polygon is a Nonagon, the triangle lmr will be equilateral and $mr = \sqrt{1-x} = ml = mn - rs = x - y$, which proves Prop. IV.

If the polygon is a Decagon, we may easily prove that the triangle Ars is isosceles; therefore $Ar = rs$. But by Prop. II, $Ar = \sqrt{1 - rs} = \sqrt{1 - Ar}$. Or putting z for Ar , $z = \sqrt{1 - z}$ = one side of a Decagon inscribed in a circle whose radius is *one*.

Cor. If we put $R=AB$, the functions $\sqrt[4]{1-x}$, $\sqrt[4]{1-y}$ and $\sqrt[4]{1-z}$, in the above cases, become $\sqrt[4]{(R^2-Rx)}$, $\sqrt[4]{(R^2-Ry)}$ and $\sqrt[4]{(R^2-Rz)}$.